practice, however, it is usually more convenient to speak of the *energy* coming to a surface each second, rather than the number of photons received.

The luminosity of an object is the total number of photons given off by the object times the energy of each photon. Instead of talking about how the number of photons must spread out to cover the surface of a sphere (brightness), we now talk about how the *energy* carried by the photons must spread out to cover the surface of a sphere. When speaking of brightness in this way, we mean the amount of energy falling on a square meter in a second. If $L$ is the luminosity of the bulb, then the brightness of the light at a distance $r$ from the bulb is given by

$$\text{Brightness} = \frac{\text{Energy radiated per second}}{\text{Area over which energy is spread}} = \frac{L}{4\pi r^2}.$$

Before moving on we offer the following aside. Usually the only information that astronomers have to work with is the light from a distant object. For this reason we will use our understanding of radiation over and over again throughout our journey. Time spent now thinking carefully about the electromagnetic spectrum, emission and absorption of photons, Planck radiation, and the inverse square law for brightness will be a *very* good investment for what is to come.

## 4.7 Radiation Laws Allow Us to Calculate the Equilibrium Temperatures of the Planets

We began our discussion of thermal radiation by asking a straightforward question: "Why does a planet have the temperature that it does?" In a qualitative way we said that the temperature of a planet is determined by a balance between the amount of sunlight being absorbed and the amount of energy being radiated back into space. We now have the tools we need to turn this qualitative idea into a real prediction of the temperatures of the planets.

Begin with the amount of sunlight being absorbed. The amount of energy absorbed by a planet is just the area of the planet that is absorbing the energy times the brightness of sunlight at the planet's distance from the Sun. When we look at a planet, we see a circular disk with a radius equal to the radius of the planet. The area of this circular disk is $\pi R^2$, where $R$ is the radius of the planet. We found in our discussion in Section 4.6 that the brightness of sunlight at a distance $d$ from the Sun is equal to the luminosity of the Sun ($L_\odot$ in watts) divided by $4\pi d^2$. (This $d$ is the same as the $r$ in the previous section. We use $d$ here to avoid confusion with the planet's radius, $R$.) We must consider one additional factor. Not all of the sunlight falling on a planet is absorbed by the planet. The fraction of the sunlight that is reflected from a planet is called the **albedo**, $a$, of the planet. The corresponding fraction of the sunlight that is absorbed by the planet is 1 minus the albedo. A planet with an albedo of 1 reflects all the light falling on it. A planet that absorbs 100 percent of the sunlight falling on it has an albedo of 0.

Writing this as an equation, we say that

$$\begin{matrix}\text{Energy absorbed}\\\text{by the planet}\\\text{each second}\end{matrix} = \begin{pmatrix}\text{Absorbing}\\\text{area of}\\\text{planet}\end{pmatrix} \times \begin{pmatrix}b = \text{Brightness}\\\text{of sunlight}\end{pmatrix} \times \begin{pmatrix}\text{Fraction}\\\text{of sunlight}\\\text{absorbed}\end{pmatrix} = \pi R^2 \times \frac{L_\odot}{4\pi d^2} \times (1 - a)$$

where $a$ is the albedo of the planet.

Moving to the other piece of the equilibrium, the amount of energy that the planet radiates away into space each second is just the number of square meters of surface area that the planet has times the power radiated by each square meter. The surface area for the planet is given by $4\pi R^2$. Stefan's Law tells us that the power radiated by each square meter is given by $\sigma T^4$. So we can say that

$$\begin{pmatrix}\text{Energy radiated by}\\\text{planet per second}\end{pmatrix} = \begin{pmatrix}\text{Surface area}\\\text{of planet}\end{pmatrix} \times \begin{pmatrix}\text{Energy radiated by}\\\text{each m}^2\text{ each second}\end{pmatrix} = 4\pi R^2 \times \sigma T^4.$$

If the planet's temperature is to remain stable—if it is to keep from heating up or cooling off—then it must be radiating away just as much energy into space as it is absorbing in the form of sunlight, as indicated in **Figure 4.27**. That means that we can equate these two expressions. We can set the quantity "Energy radiated by planet" equal to the quantity "Energy absorbed by planet." When we do this, we arrive at the expression

$$\begin{matrix}\text{Energy radiated by}\\\text{the planet each second}\end{matrix} = \begin{matrix}\text{Energy absorbed by}\\\text{the planet each second}\end{matrix}$$

or

$$4\pi R^2 \sigma T^4 = \pi R^2 \frac{L_\odot}{4\pi d^2}(1 - a).$$

Look at this equation for a moment. It may seem rather complex, but when broken into pieces it becomes more digestible. On the left side of the equation, $4\pi R^2$ tells how many square meters of the planet's surface are radiating energy back into space, while $\sigma T^4$ tells how much energy

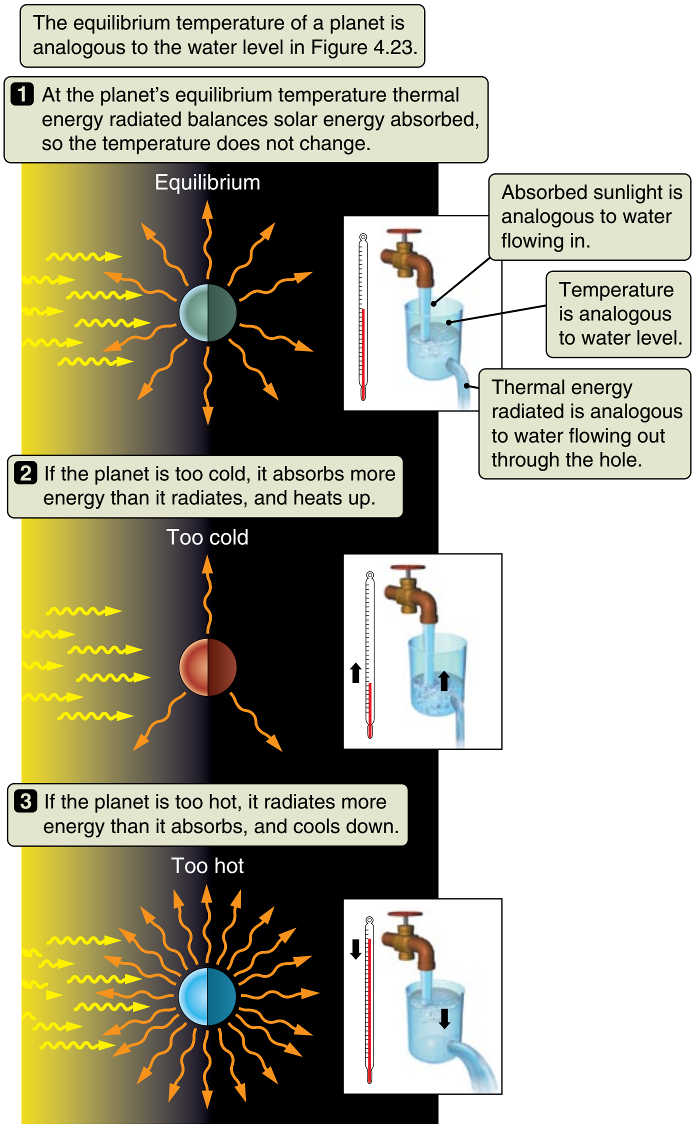


**FIGURE 4.27** Planets are heated by sunlight and cooled by emitting thermal radiation into space. If there are no other sources of heating or means of cooling, then the equilibrium between these two processes determines the temperature of the planet.

each one of those square meters radiates each second. Put them together, and you get the total amount of energy radiated away by the planet each second. On the right side of the equation, $\pi R^2$ is the area of the planet as seen from the Sun. That amount times the brightness of the sunlight reaching the planet, $L_\odot/4\pi d^2$, tells how much energy is falling on the planet each second. The final $1 - a$ tells how much of that energy the planet actually absorbs. Put everything on the right side of the equation together, and you get the amount of energy absorbed by the planet each second. The equal sign says that the energy radiated away needs to balance the sunlight absorbed. There is no magic here. In fact, when broken down, this formidable equation embodies little more than a few straightforward ideas such as "hotter means more luminous," "twice as far means one-fourth as bright," and "heating and cooling must balance each other." The math just gives us a convenient way to work with these concepts.

We started down this path hoping to find a way to predict the temperatures of the planets, and a bit of algebra gets us the rest of the way there. Rearranging the previous equation to put $T$ on one side and everything else on the other gives

$$T^4 = \frac{L_\odot(1-a)}{16\sigma\pi d^2}.$$

If we take the fourth root of each side, we wind up with

$$T = \left[\frac{L_\odot(1-a)}{16\sigma\pi}\right]^{\frac{1}{4}} \times \frac{1}{\sqrt{d}}.$$

We have now produced a full-fledged physical model for why the temperatures of planets are what they are. Restating the meaning in words, $T$ is the temperature at which the energy radiated by a planet exactly balances the energy absorbed by the planet. If the planet were hotter than this equilibrium temperature, it would radiate energy away faster than the planet absorbed sunlight, and the temperature would fall. If the planet were cooler than this temperature, it would radiate away less energy than was falling on it in the form of sunlight, and the temperature of the planet would rise. Only at this equilibrium temperature do the two balance.

**Balancing cooling and heating sets an equilibrium temperature.**

The equation tells us that as the distance from the Sun increases—in other words, as $d$ gets bigger—the temperature of the planet decreases. No surprise there. But now we know how much the temperature should decrease. It should be inversely proportional to the square root of the distance. Using this formula can turn our intuition about why planets that are close to the Sun are hot into a prediction of just how hot they should be.

**Figure 4.28** shows a graph of the predicted temperatures of the planets. The vertical bars show the range of temperatures found on the surfaces of each planet (or, in the case of the giant planets, at the tops of their clouds). The black dots show our predictions using the equation above. From the figure, you can see that overall we are not too far off. That should give us a sense of accomplishment: It says that our basic understanding of *why* planets have the temperatures that they do is probably not too far off. Mercury, Mars, and Pluto agree particularly well. (The agreement for Mercury would improve if we took into account the huge difference in temperature between the daytime and nighttime sides of the planet and recomputed our equilibrium accordingly.)

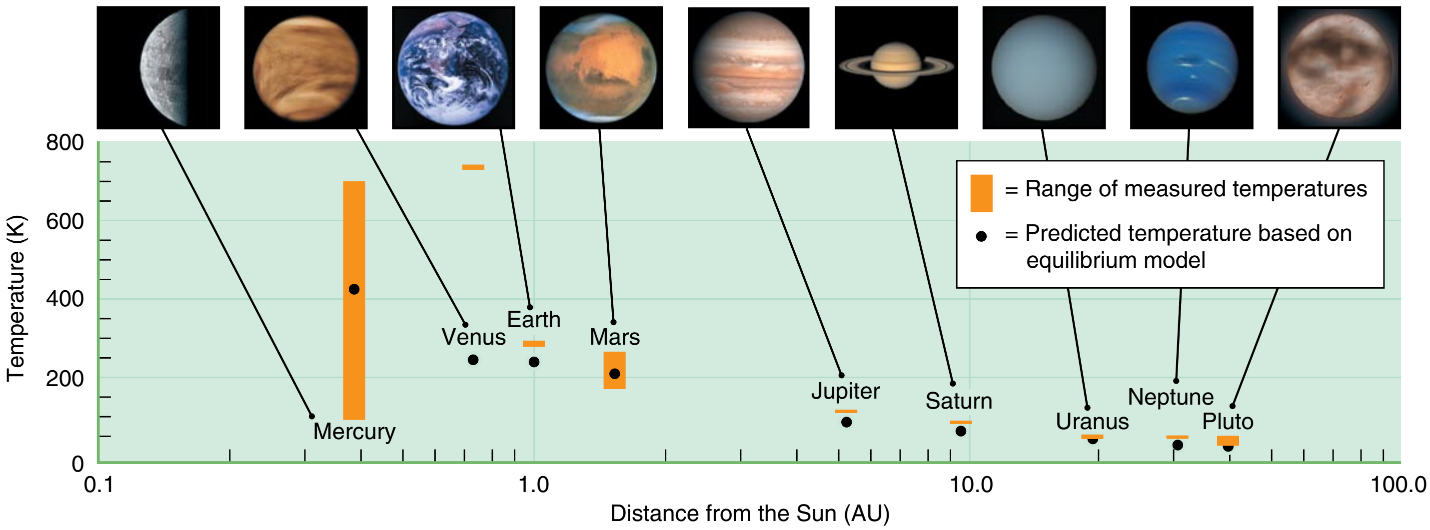


**FIGURE 4.28** Predicted temperatures for the classical planets and Pluto, based on the equilibrium between absorbed sunlight and thermal radiation into space, are compared with ranges of observed surface temperatures. Some predictions are correct. Interestingly, others are not.

In other cases, however, our predictions are wrong. For Earth and the giant planets the actual temperatures are a bit higher than the predicted temperatures. In the case of Venus the actual surface temperature is wildly higher than our prediction. Rather than cause for despair, these discrepancies between theory and observation are cause for excitement. As we built our physical model for the equilibrium temperatures of planets, we made a number of assumptions. For example, we assumed that the temperature of the planet was the same everywhere. This is clearly not true: We might expect planets to be hotter on the day side than on the night side. We also assumed that a planet's only source of energy is the sunlight falling on it. Finally, we assumed that a planet is able to radiate energy into space freely as a blackbody. The discrepancies between our theory and the measured temperatures of some of the planets tell us that for these planets, some or all of these assumptions must be incorrect. In other words, the places where the predictions of our theory are not confirmed by observation point to areas where there is something still to be discovered and understood. The question of *why* these planets are hotter than the prediction will lead us to a number of new and interesting insights into how these planets work. Scientific theories sometimes succeed and sometimes fail, but even when they fail they can teach us a lot about the universe.

# Summary

- From gamma rays to visible light to radio waves, all radiation is an electromagnetic wave.
- Light is also a stream of particles called photons.
- The speed of light in a vacuum is 300,000 km/s, and nothing can travel faster.
- Like gravity, light obeys the inverse square law.
- Light from receding objects is redshifted. Light from approaching objects is blueshifted.
- Special relativity concerns the relationship between events in space and time.
- Space and time together form a four-dimensional spacetime.
- Nearly all matter is composed of atoms.
- Atoms absorb and emit radiation at unique wavelengths like spectral fingerprints.
- Temperature is a measure of the thermal energy of an object.